

# 6.270 Lecture

Sensors, Motors, Gear Ratios,  
& Motor DC Theory

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# Overview of Lecture

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- ▶ **Sensors**

- ▶ Switches, Breakbeam/Optical Encoders, GyroScopes,
- ▶ Pull-up/Down Resistors
- IR-LED & Phototransistor, & Sharp IR Distance Sensor

- ▶ **Motors**

- ▶ 6.270 DC Motor
- ▶ Servo Motor
- ▶ Continuous Servo Motor

- ▶ **Gear Ratios**

- ▶ Torque and Speed Tradeoff
- ▶ Sample Calculation/s

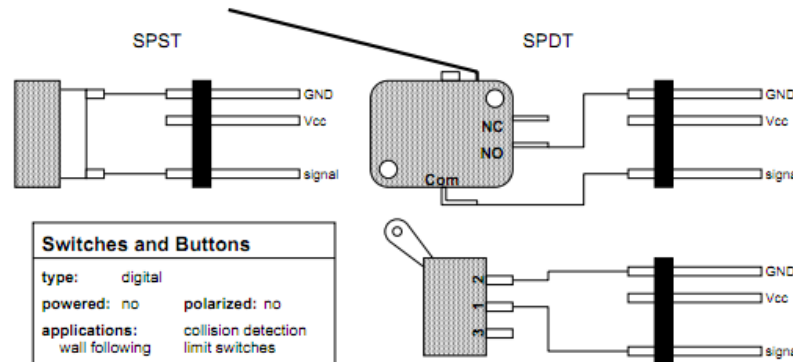
- ▶ **DC Motor Theory**

- ▶ PWM / H-bridge
- ▶ Torque vs Speed, Current, Power, and Efficiency Curves
- ▶ Sample Calculation/s



# Sensors - Switches

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- ▶ Digital Input – Only Binary outputs (1 or 0)

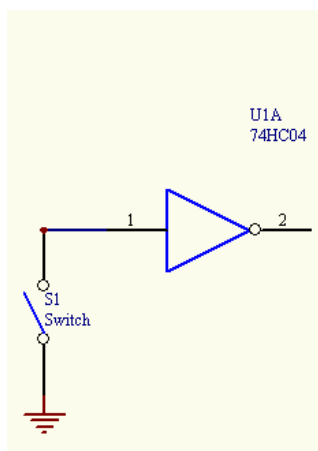
Wiring (Pins 0-23); Pins(0-7 recommended)

- ▶ Single Pole Single Throw (SPST) Single Pole Double Throw (SPDT)
- ▶ Normally Open (NO or digital output is 0 unless switch is pressed) vs. Normally Close (NC or digital output is 1 unless switch is pressed)
- ▶ Applications Wall Following, Alignment, Logic Stop
- ▶ Note about Pull-Up/Pull-Down Resistors
- ▶ Example on HappyBoard
  - ▶ `digital_read(Port #)`

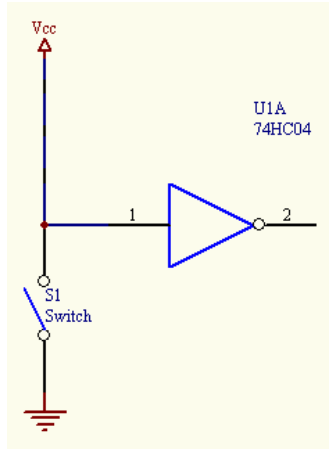
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- ▶ For Digital Inputs: Always have Pull-up Resistors ON

# Pull-Up, Pull-Down Resistors

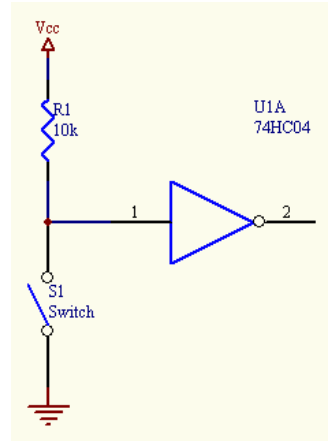
- ▶ Logic gates can have floating values (1 or 0).
  - ▶ Susceptible to Electrical Noise
- ▶ Pull-up/down Resistors always gives a definite value to logic.



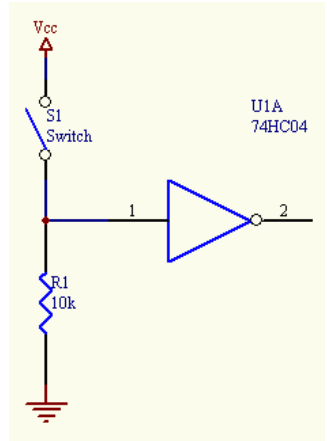
BAD: Floating Logic Gate  
Susceptible to Electrical Noise



BAD: Solves Problem until  
switch is closed leading to a  
short



GOOD – Pull-up - When Switch is  
OFF, Logic is set to HIGH

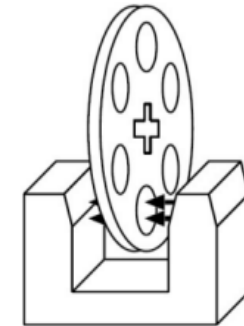
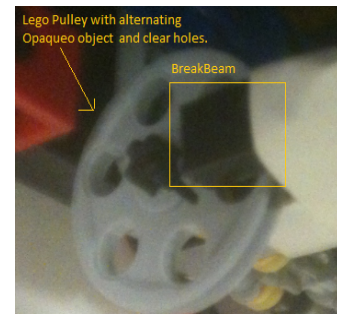
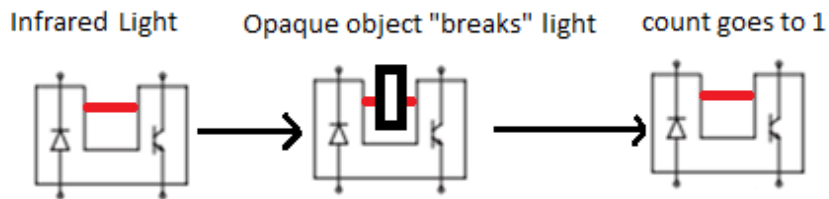


GOOD –Pull-down. When switch is  
OFF, Logic is set to LOW.

▶ Images from: <http://www.seattlerobotics.org/encoder/mar97/basics.html>

# Sensors:

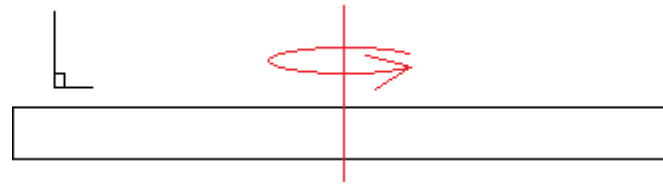
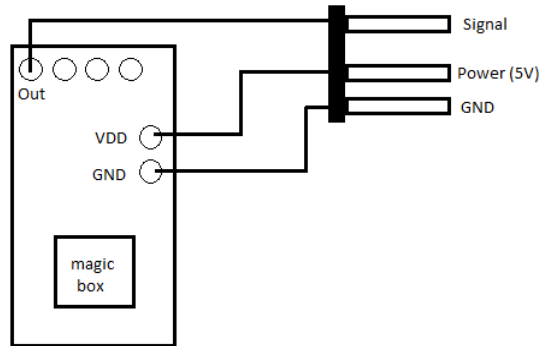
## BreakBeam/Optical Shaft Encoders



- ▶ Encoder Input (24-27): Transitive analog/Digital
  - ▶ IR-LED on Left, Phototransistor on right
- ▶ Wiring Tips
  - ▶ Look for Diode Symbol
  - ▶ GNDs are Diagonal from each other.
- ▶ Applications – Track # of Wheel Rotations. Distance Calculation
- ▶ HappyTest Example
  - ▶ `encoder_read(port#)` and `encoder_reset(port#)`



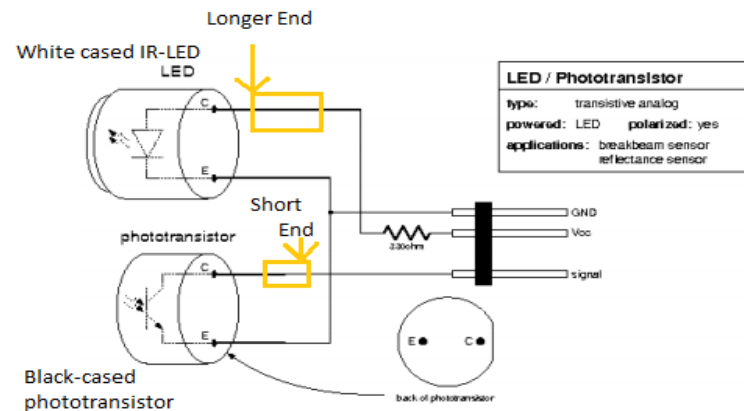
# Sensors: Gyroscopes



- ▶ Analog (8-23)
  - ▶ Recommended (20-23) with Pull-Up resistors to OFF
- ▶ Application: Measures the *perpendicular Axis of Rotation* using Velocity Integration:
  - ▶ Constant of Integration builds up error over time  $x = \int v dt + C$
- ▶ Needs Calibration of Rotation Angle Multiplier
- ▶ HappyBoard Example
  - ▶ `gyro_init (GYRO_PORT, LSB_US_PER_DEG, 500L);`
    - ▶ Robot should be stationary during calibration
  - ▶ `gyro_get_degrees()` returns float

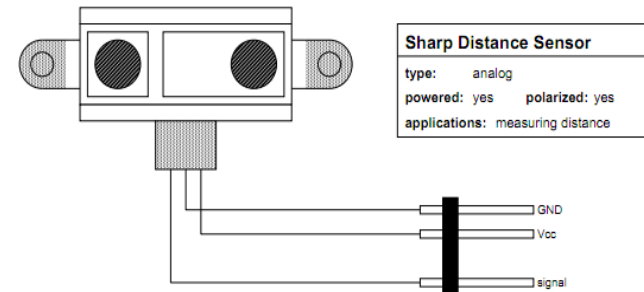
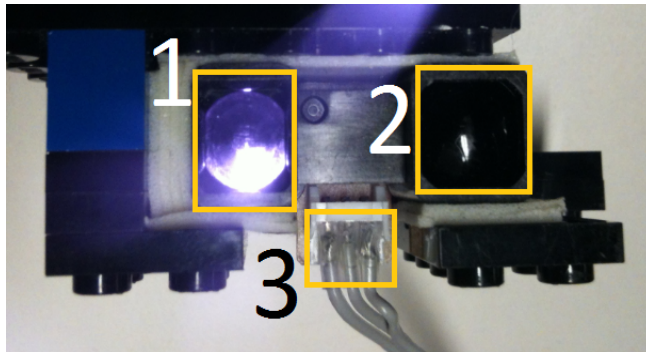


# Sensors: IR-LED + Phototransistor



- ▶ Analog Input(0-23): For 20-23, have the Pull-ups ON
- ▶ InfraRed is emitted from LED. Phototransistor Receives light.
  - ▶ More Light = Lower Resistance = Lower analog Value
- ▶ Applications: BreakBeam, Line Follower, Light Follower
- ▶ HappyTest Example:
  - ▶ `analog_read(port#)`

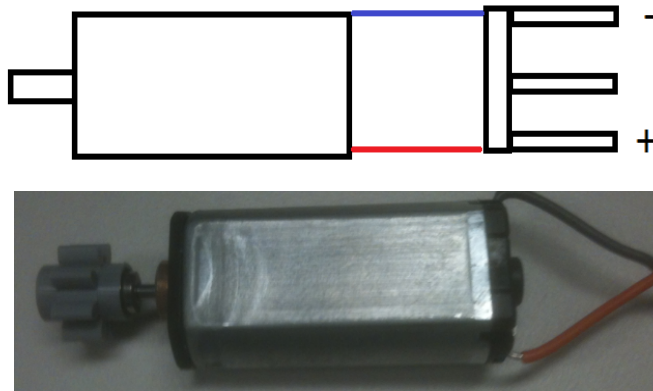
# IR + Sharp Distance Sensor



- ▶ Applications – measures distances from 8”-60”
  - ▶ Doesn't have to be perpendicular.
- ▶ Analog Input, pins **20-23 only**. Pull-Up Resistors are OFF. (See HappyLab for 'loophole')
  - ▶ Sensor provides its own analog input.
- ▶ Near-Infrared is emitted from #1; #2 Measures Angle.
- ▶ Needs Calibration
  - ▶ Non-linear method of measuring distances
- ▶ HappyBoard Example: `irdist_read(Port#)` returns float in cm



# Motors: DC Motor



	Reflected Values	
Free Speed	14292	RPM
Free Current	0.39	Amps
Stall Current	2.2	Amps
Stall Torque	0.00587	N-m
Internal R	3.181818182	

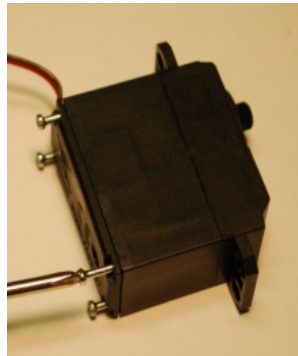
Motor Operating V: 7

Kt(Torque Constant)	0.003241	Nm/A
Kv (Velocity Constant)	1938.6	RPM/V
ke(Back emf)	0.003241	V/rad/s

- ▶ Motor Pins 0-5 – Ask Organizers for extra Motor Drivers
- ▶ Must have 8-tooth Gear Connected
- ▶ Very High RPM, Very Low Torque
  - ▶ Useless **unless** Gear Ratios are used to Increase Torque and Decrease Speed.
- ▶ POS, NEG terminals no distinct PWR/GND
  - ▶ Note: Flipping Connection flips Motor Direction
- ▶ HappyBoard Example: `motor_set_vel(Port#, Speed)`
  - ▶ Speed Ranges from -255 to 255. 0 is stop.

# Motors- Positional Servos

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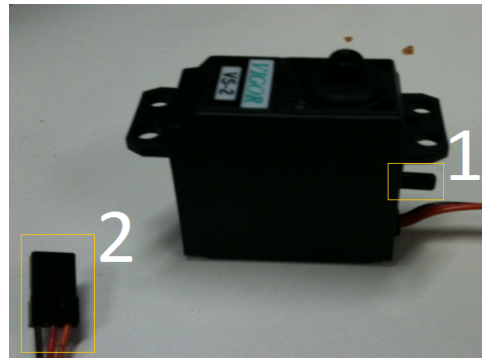


- ▶ Servo Pins 0-5. Three Cables: GND, PWR, SIG
- ▶ Precise Actuator limited to 0-180 degrees.
  - ▶ Actively set angular position of servo
  - ▶ No gear ratios
- ▶ Low RPM, Very High Torque
- ▶ Applications – Slow & powerful arms, precise open-loop motions
- ▶ HappyBoard Example: `servo_set_pos(port#, pos)`. Pos ranges from 0-511. CAREFUL with Extreme Positions.



# Motors: Continuous Servos

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- ▶ No longer restricted to 0-180deg, but no longer capable of precise motions.
  - ▶ Essentially a High Torque, Low RPM DC motor.
- ▶ **NOT Recommended. Positional Servos are *beautiful!***
  - ▶ Permanent Change. Potential Screw-up
- ▶ Potentiometer Calibration is needed -> Servo's Center changes.  
Use HotGlue to find center
- ▶ HappyBoard Example:
  - ▶ Potentiometer's effect on: `servo_set_pos(port#, pos)`  
`servo_disable(port#)`



# Gear Ratios

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- ▶ Concept: Change output angular velocity & its Torque using gears

- ▶ Important Equation:  $Gear\ Reduction = \frac{N_{out}}{N_{in}}$

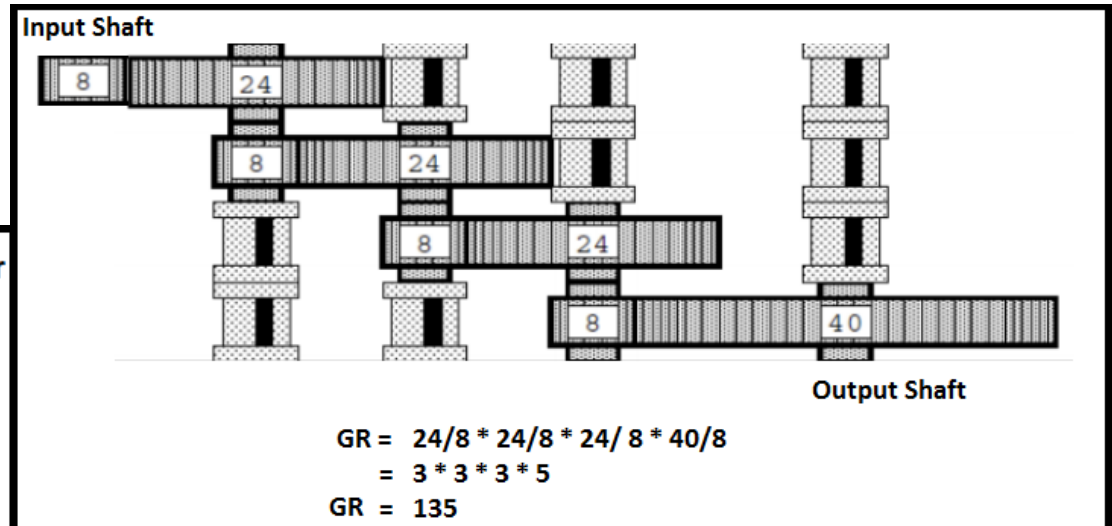
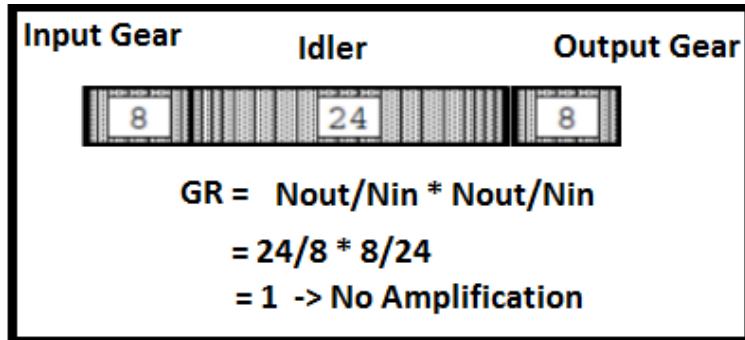
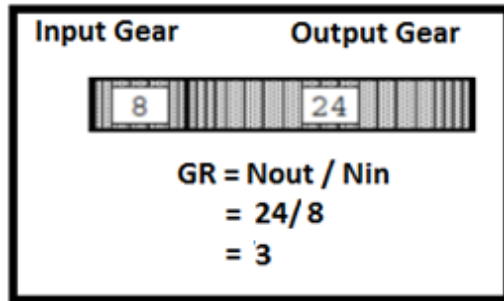
- ▶ Derivables:  $Torque\ Ratio = G.R. = \frac{T_{out}}{T_{in}}$        $Speed\ Ratio = \frac{W_{in}}{W_{out}}$

$$\frac{N_{out}}{N_{in}} = \frac{W_{in}}{W_{out}} = \frac{T_{out}}{T_{in}}$$

- ▶ Mechanical Advantage attained through conservation of Power  $P = T_A\omega_A = T_B\omega_B$ ,      Which yields  $MA = \frac{T_B}{T_A} = \frac{\omega_A}{\omega_B}$ .
- ▶ Higher Gear Reduction amplifies torque, trading speed.
- ▶ Lower Gear Reduction amplifies speed, trading torque.



# Gear Ratios – 10% Loss each Stage



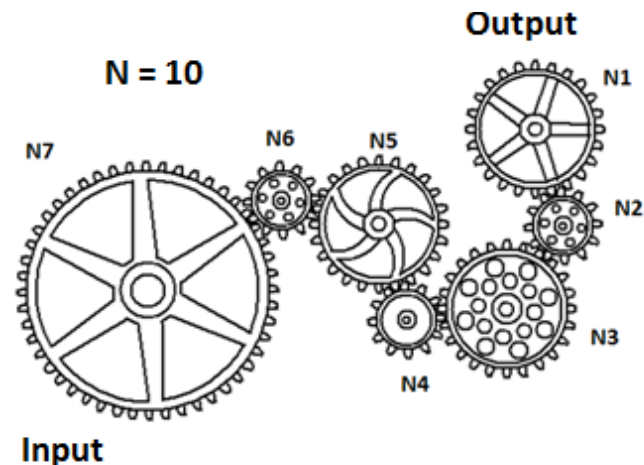
- ▶ Simplest Case use:  $N_{out} / N_{in}$
- ▶ Compound Gearing requires Repeated Multiplication of Gear Ratios at Every Stage
- ▶ Idler Gears – Intermediate Gear does not contribute to ratio
  - ▶ Only Input and Output Matters
- ▶ Useful for switching directions and Spacing

# Gear Ratio Sample Calculations

- ▶ Given: Motor's Torque = 2N-m & RPM = 1000
- ▶ Calculate the Following Gear Ratios
- ▶ Which ones amplify torque and which amplifies speed?
  - ▶ By How Much?

▶ Remember: *Gear Reduction* =  $\frac{N_{out}}{N_{in}}$

$$\frac{N_{out}}{N_{in}} = \frac{W_{in}}{W_{out}} = \frac{T_{out}}{T_{in}}$$

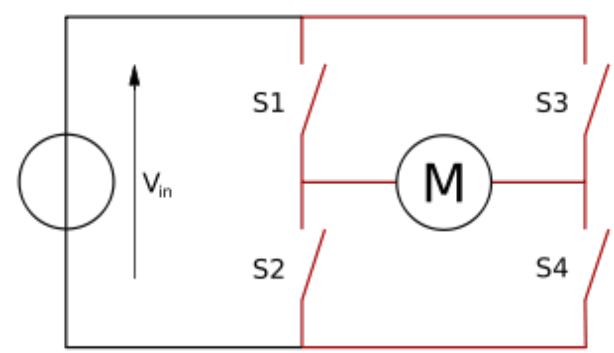


ANS: Left: GR = 15. Geared for Torque. Output RPM: 1000/15; Output Torque: 30Nm

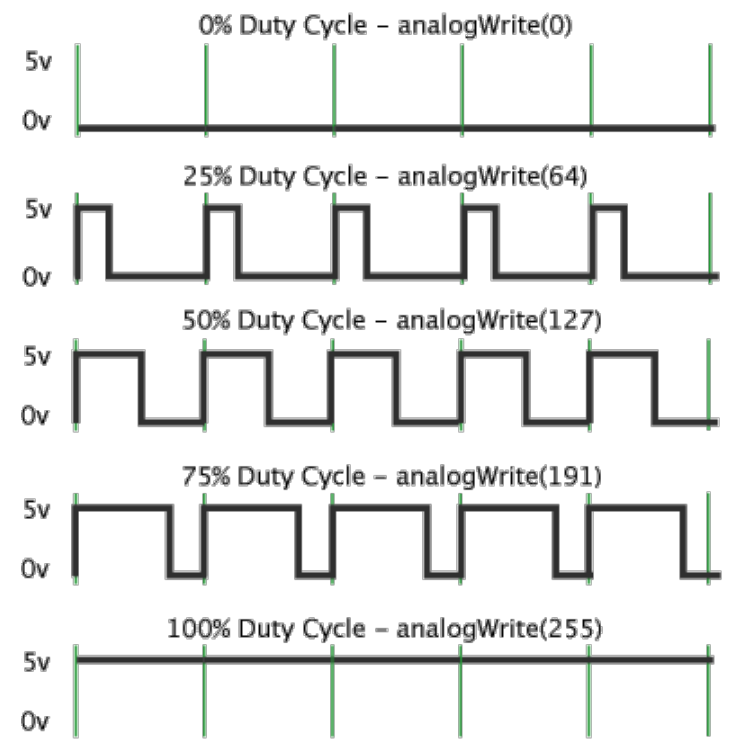
ANS: Right: GR = 1/7. Geared for Speed. Output RPM: 7000; Output Torque: 2/7Nm

# DC Motor Theory – PWM and H-Bridges

- ▶ PWM – Pulse Width Modulation
- ▶ Changes Voltage across Battery
  - ▶ Scales RPM, Torque, Proportionally
  - ▶ No magical torque comes out when RPM goes down!
- ▶ H – Bridges – Directional Control
- ▶ of Motor



Pulse Width Modulation



<http://arduino.cc/it/Tutorial/PWM>



# DC Motor Theory

Gear Ratio:	3					
	Reflected Values	Effective Gear Ratio		Motor Operating V:	7	
Free Speed	14292	4764	RPM			
Free Current	0.39	0.39	Amps			
Stall Current	2.2	2.2	Amps			
Stall Torque	0.00587	0.01760	N-m	Kt(Torque Constant)	0.003241	Nm/A
Internal R	3.181818182			Kv (Velocity Constant)	1938.6	RPM/V
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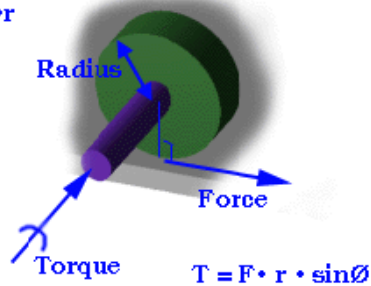
- ▶ Everything in DC Motor linearly Scales.
- ▶ Torque vs Speed Relationship

$$\tau_{\text{motor}} = \tau_s - \omega \tau_s / \omega_n$$

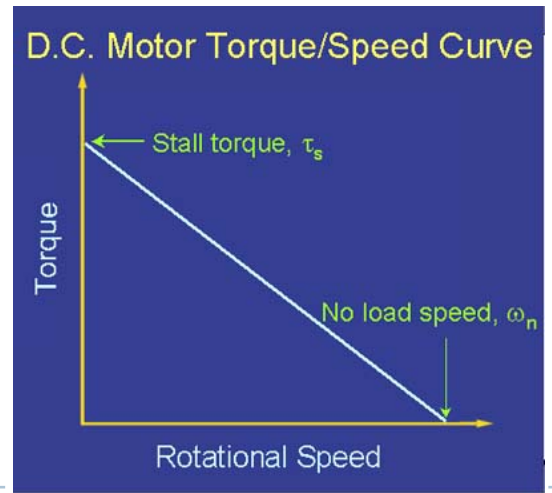
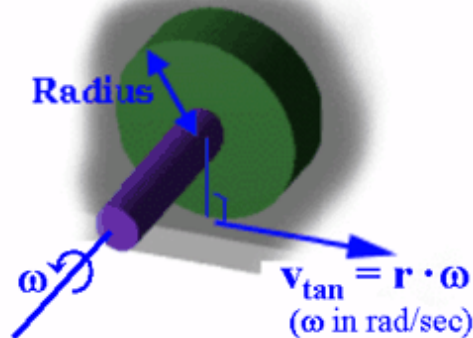
$$\omega_{\text{motor}} = (\tau_s - \tau) \omega_n / \tau_s$$

For the case of a wheel or winch the force is always tangent.

$$T = F \cdot r$$



## Angular Velocity

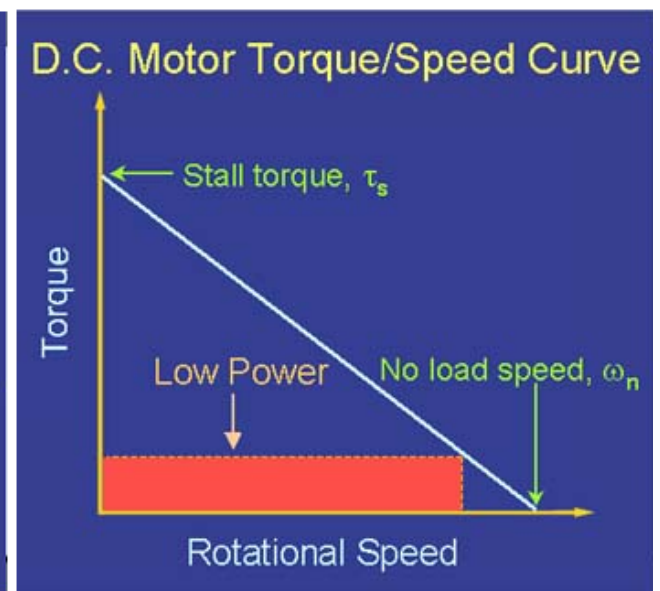
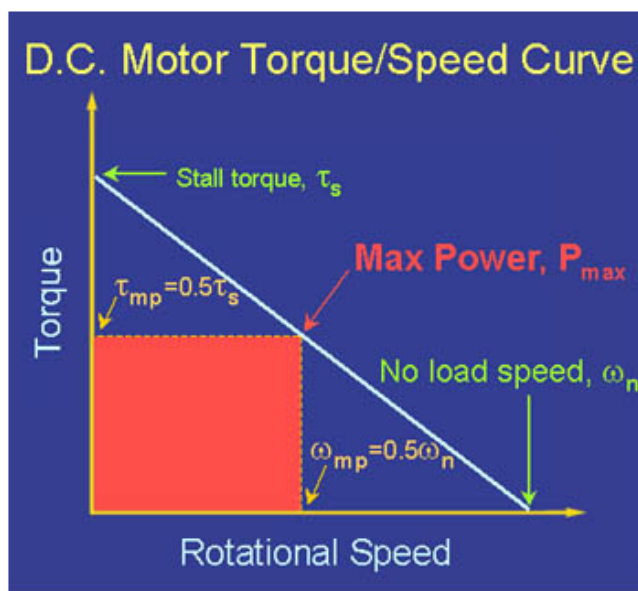
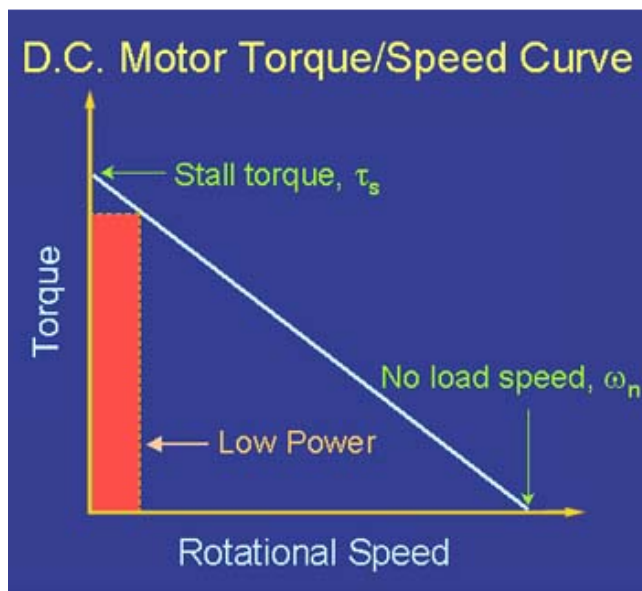


▶ <http://lancet.mit.edu/motors/motors3.html>



# DC Motor Theory

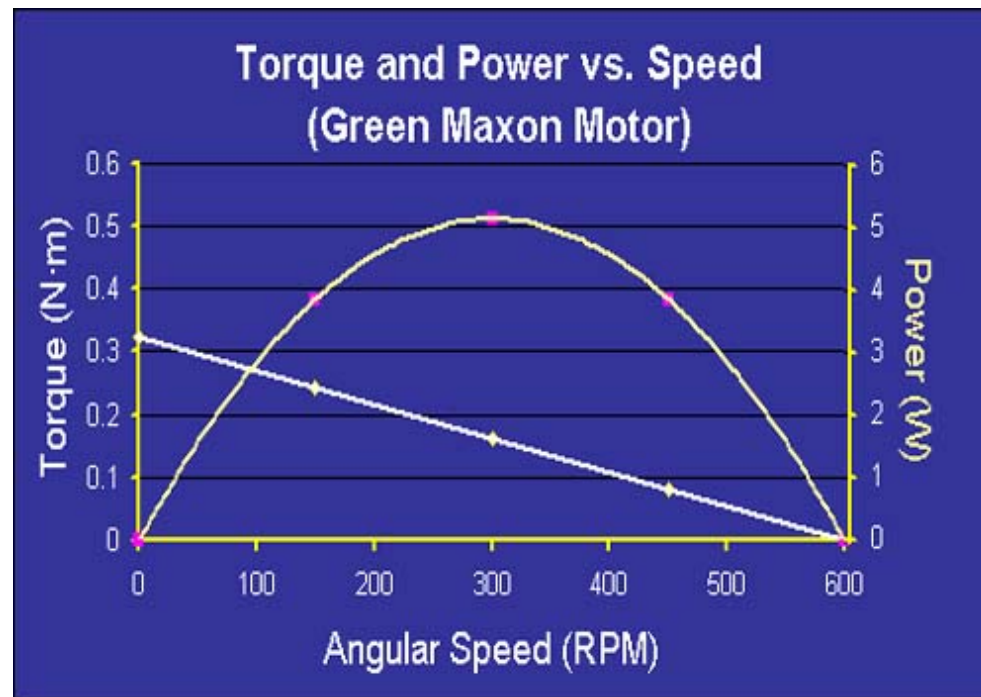
- ▶ Power = Torque \* Rotational Velocity (in radians)
- ▶ Max Power Occurs at  $\frac{1}{2}$  Stall Torque and  $\frac{1}{2}$  Rotational Speed



# DC Motor Theory

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- ▶ Plotting Torque & Power vs Speed
  - ▶ Further shows that Max Power occurs at  $\frac{1}{2}$  Torque and  $\frac{1}{2}$  Speed



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- ▶ <http://lancet.mit.edu/motors/motors3.html>

# DC Motor Theory – Usefulness?

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## ▶ Remember Gear Ratios?

- ▶ 6.270 DC Motors are very high RPM and very low Torque.
- ▶ Gear Ratios scale
  - ▶ Use G.R. to amplify torque outputs
  - ▶ When Torque Increases, Velocity Decreases

Gear Ratio:	3					
	Reflected Values	Effective Gear Ratio		Motor Operating V:	7	
Free Speed	14292	4764	RPM			
Free Current	0.39	0.39	Amps			
Stall Current	2.2	2.2	Amps			
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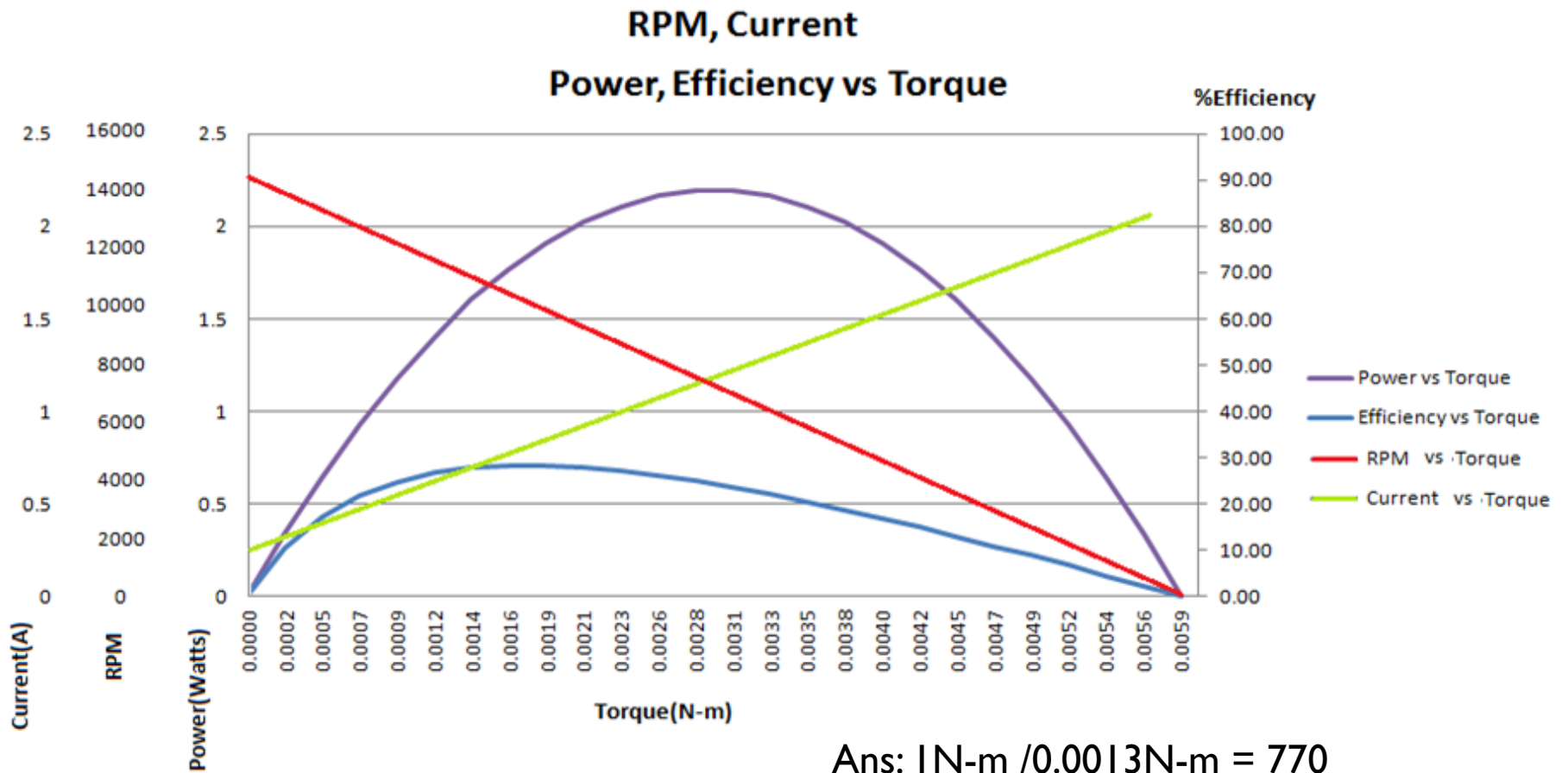
## ▶ Other Usefulness:

- ▶  $K_t$  : Nm/A : How much Torque is provided for a given current passing through Motor
- ▶  $K_v$  : RPM of motor for given V



# DC Motor Theory

- ▶ Useful Visualization: Given a Required Torque of 1N-m, Find a Gear Ratio that operates near Efficiency



Ans: 1N-m / 0.0013N-m = 770

Gear Ratio: 770:1 → A bit Ridiculous.

# DC Motor Theory

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Free Speed	14292
Free Current	0.39
Stall Current	2.2
Stall Torque	0.00587
Motor Operating V:	7

- ▶ Everything you need to know about a Motor's characteristics ( $K_t$ ,  $K_v$ ,  $K_e$ , Efficiency, Power, Graph, etc) can be derived from these scalar values
- ▶ Be sure to Download the 6.270 DC Motor Excel Graph to see how the math works.
- ▶ For the Lazy: Simply Input Torque and you Get Everything

	Torque Input(N-m)	vs RPM	vs Current(A)	vs Power(W)	vs %Efficiency
Input Torque	0.005	2110.7	1.9	1.1	8.2
At Max Power	0.00293	7146.0	1.3	2.2	24.2

